Proceedings of
2001 ASME International Mechanical Engineering Congress and Exposition
November 11–16, 2001, New York, NY

DESIGN AND SIMULATION OF SHAPED COMB FINGERS FOR COMPENSATION OF MECHANICAL RESTORING FORCE IN TUNABLE RESONATORS

EXTENDED ABSTRACT

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INTRODUCTION

The comb-drive actuator is one of the main building blocks of microelectromechanical systems (MEMS). Its working principle is based on an electrostatic force that is generated between biased conductor plates as one moves relative to the other. Because of its capability of force generation, it finds wide application in micro-mechanical systems. Sample applications include polysilicon microgrippers [1], scanning probe devices [2], force-balanced accelerometers [3], actuation mechanisms for rotating devices [4], laterally oscillating gyroscopes [5], and RF filters [6]. Consequently, any improvement to this basic actuator could have far-reaching effects.

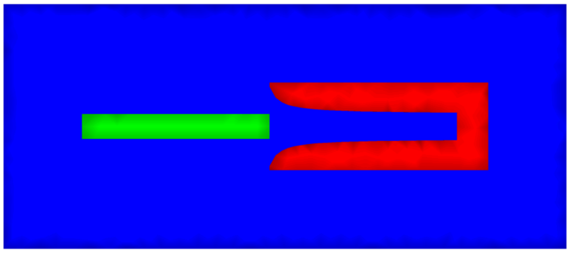
Specifically, we are interested in shaped comb finger designs which would generate force-deflection profiles that have linear shapes. These linear relationships could partially compensate for the mechanical restoring force due to the action of a linear suspension spring. This electrostatic weakening or stiffening of the mechanical spring can decrease the drive voltage of actuators or change the resonant frequency of resonators.

Several previous researchers have investigated various comb shapes. Hirano et al. [7] reported techniques for fabricating fingers which could dramatically reduce the separation gap after only a short motion. These fingers were designed for maximum possible force output at a nearly constant rate. Rosa et al. [8] continued this search for high-force actuators by designing and testing actuators with angled comb fingers. Ye et al. [9] studied directly the force-deflection behavior of a number of finger designs using a two-dimensional numeric electrostatic solution. They reported designs with linear, quadratic, and cubic behavior. This work focuses on designing, modeling, and testing of shaped comb fingers with linear force profiles for use in tunable resonators. A tunable resonator designed using the principles outlined here has been designed with stiffness tuning of up to 50% at 100 V tuning voltage, compared to 4.9% stiffness tuning (for a weaker spring) in an earlier work [6]. This extended abstract describes the key points of the work.

COMB FINGER MODELING

First, a simple model for force-deflection behavior of shaped comb fingers is derived. The result, based on the physics of the system, is

$$F_x = \frac{V^2 \varepsilon_0 t}{g(x)} \tag{1}$$



Shaped Design

Figure 1: Top views of two finger designs modeled using CoventorWare.

where F_x is the force pulling on the moving comb finger, V is the voltage between fingers, ε_0 is the permittivity of free space, t is the out-of-plane thickness of the fingers, x is a coordinate describing how much the fingers are engaged, and $g(x)$ is a function describing the gap between the fingers. Eq. (1) is valid only if either the fixed or the moving finger is rectangular, with the other finger assuming the shape which results in the gap profile $g(x)$. The outstanding feature of this model is its prediction that the force-deflection behavior of the finger is proportional to the reciprocal of the gap profile.

FINITE ELEMENT MODELING

To further test the simple model of Eq. (1), a sample finger design was simulated using CoventorWare, a MEMS simulation tool. A top view of the finger design is included in Fig. 1. The gray rectangle under the fingers is the substrate, which acts as a ground plane during simulation. The finger design is based on the model of Eq. (1), with a gap profile equal to the reciprocal of a linear function. The simple model predicts that such a shape will produce linear force-deflection behavior. The modeled comb finger was simulated over a 20-μm engagement range (from 5 μm engagement to 25 μm) to determine the electrostatic force acting on the fingers as a function of displacement.

MODELING RESULTS

A graph showing the response for the shaped finger is plotted in Fig. 2. In this graph, the least-squares polynomial fit to the simulation data and results from the simple model are also presented. As the graph shows, the shaped fingers have a linear force/displacement trend. However, the simple model predicts larger force than the simulation results. Which of these predictions is more accurate will be considered later. At this point, it is sufficient to know that the finger shape does give a linear force-engagement curve, as desired.

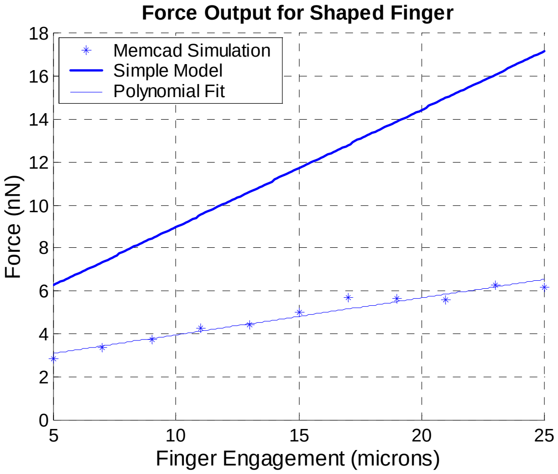


Figure 2: Force output of shaped finger design.

SHAPED FINGERS IN A TUNABLE RESONATOR

The linear behavior of the shaped fingers allows design of a tunable resonator with well-predicted tuning capability. This is done by modifying a standard comb resonator to include one or more banks of shaped comb electrodes whose potential can be controlled independently of the normal drive and sense electrodes. The resonant frequency of such a system can then be derived, to first order, as

$$\omega = \sqrt{\frac{k_m - V^2 n k_t}{m}} \qquad (2)$$

where V is the DC voltage applied to the shaped tuning electrodes, n is the number of shaped fingers, and k_t is the factor describing the magnitude of the linear response of one shaped finger. ω is the resonant frequency of the system, k_m is the mechanical spring constant of the resonator, and m is the resonator mass.

TUNABLE RESONATOR SIMULATION AND TESTING

Based on the good results from the initial simulations, shaped comb fingers were designed to be incorporated in tunable resonators. Two designs were generated: one which would weaken or reduce the effective stiffness of the resonator, thereby reducing resonant frequency, and one which would increase the effective stiffness. The weakening resonator layout is shown in Fig. 3. The resonators were designed and fabricated using the SUMMiT technology, a four-level polysilicon surface micromachining technology developed at Sandia National Laboratories. By patterning the stacked layers of polysilicon into the desired finger shapes, a finger may be produced with an increased out-of-plane thickness, as illustrated in the comb finger model also shown in Fig. 3. This figure shows the CoventorWare model of the shaped fingers used for the weakening resonator. Again, the large rectangle under the fingers is the substrate, which acts as a ground plane. A micrograph of the fabricated shaped comb fingers is also presented in Fig. 3. In this picture, the rectangular

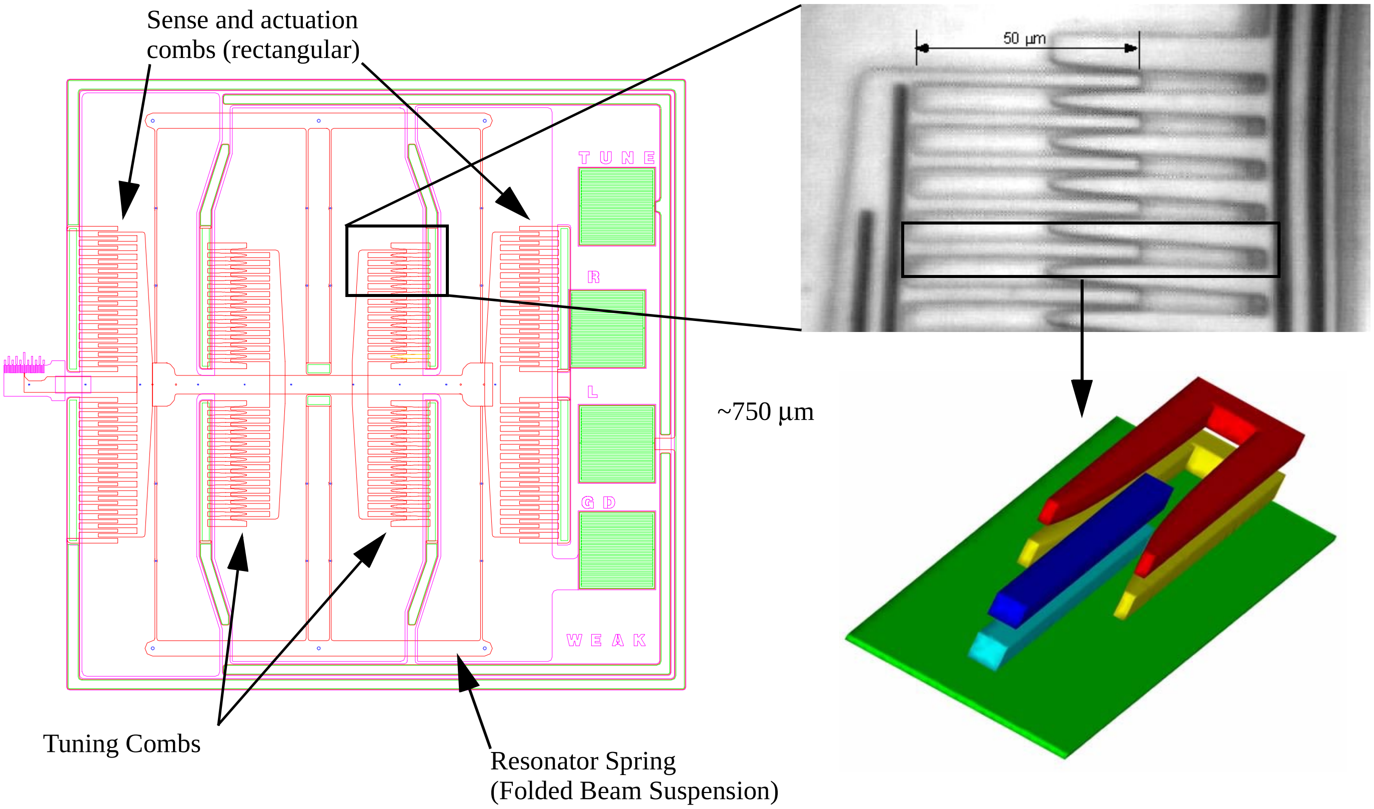


Figure 3: Layout of a tunable resonator with decreasing resonant frequency with increasing DC tuning voltage. A microscope photograph of the tuning combs and the simulation model of the tuning fingers are also shown.

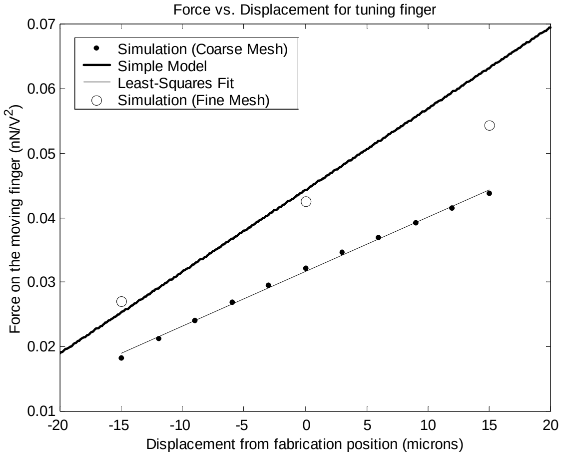


Figure 4: Force-Deflection predictions for the weakening finger shape.

fingers (on the left) are movable, while the shaped fingers (on the right) are stationary.

Fig. 4 shows the simulation results compared to the simple model for the weakening fingers. As before, the trend is linear, with a smaller force output and slope than the simple equation predicts. However, a few simulations performed using a finer mesh give results much closer to those of the simple model, implying that the simple model predicts behavior well. The prohibitive length of time required to generate data points at the finer mesh—about 14 hours per data point, compared to approximately 30 minutes per data point at the coarser mesh—discourages full simulation at this mesh scale, though. The accuracy of the simple model is further indicated by Fig. 5, which shows experimental data for a tuned weakening resonator. The resonant frequency of the device is measured using the blur envelope technique, in which the resonant frequency corresponds to the frequency which produces the largest blur envelope, and thus the largest deflection of the resonator. The resolution of this technique is about ±50 Hz. The measured data agree extremely well with the prediction of the simple model, which is Eq. (2), with k_t predicted using Eq. (1). The simulation results, on the other hand, predict tuning behavior poorly. The prediction as well as the experiment shows that this resonator design allows tuning from the base frequency of 4.36 kHz down to near 0 Hz over a DC tuning range of 0-80 V. Testing of the stiffening resonator was also successful, with an increase from 4.36 kHz to 5.2 kHz over a DC tuning range of 0-80 V.

Hence, the fabricated devices demonstrate the use of shaped comb banks to tune the resonant frequency of a resonator. These designs were not optimized for a particular application; rather

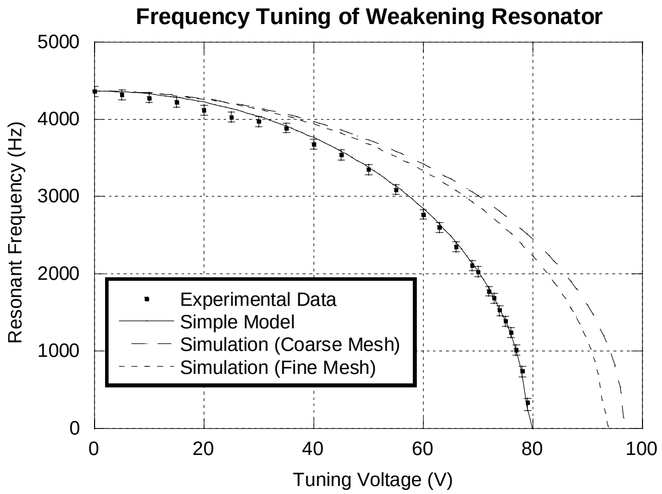


Figure 5: Experimental data compared to the model predictions for tuning of a resonator

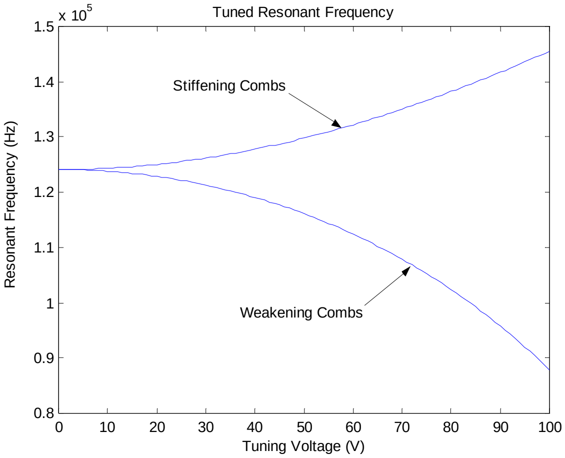


Figure 6: Predicted tuning range for a new resonator design.

they were meant to allow testing of the concept. For example, Fig. 6 shows predicted tuning behavior for a much stiffer resonator with both weakening and stiffening combs. Resonant frequency can be selected anywhere between 145 kHz and 87 kHz with up to 100 V tuning voltage. This represents stiffness tuning from a stiffness of about 496 N/m up to about 677 N/m (+36.5%) or down to 244 N/m (-50.8%). By comparison, previous work has demonstrated stiffness tuning from approximately 143 N/m down to 136 N/m (-4.9%) over a 100 V tuning range using parallel-plate tuning [6]. Note also that tuning using parallel plates allows only reduction of stiffness; augmentation of stiffness is not possible. In addition to resonator tuning, shaped fingers could be applied to the design of low-voltage actuators, in which a part of the actuated load would be offset by the tuning combs.

ACKNOWLEDGEMENTS

The assistance of Ming Yu in gathering experimental data is gratefully acknowledged. The staff of the Microelectronics Development Laboratory at Sandia National Laboratories are also gratefully acknowledged for their fabrication work. This work is supported in part by a Nation Science Foundation Graduate Research Fellowship and a National Science Foundation CAREER Award. Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy under contract DE-AC04-94AL85000.

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